The Cost of Inflation

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Inflation and relative price deviation

- Inflation can cause relative prices to deviate from the general equilibrium values defined by the ‘real primitives’ of the economy
- Deviation of relative prices from their general equilibrium values generates resource misallocation
- How much relative price deviation does inflation cause?

General equilibrium prices not known

- General equilibrium prices are not known
- We cannot, therefore, directly measure deviations of observed prices from their general equilibrium values
- Most economists use an indirect measure of relative price deviation: ‘correlation between inflation and size of price change’

Size of price change as a proxy for relative price deviation

Under Menu-Cost (state-dependent) and Calvo style (time-dependent) pricing:

'large relative price deviation'

$$\Downarrow\Uparrow$$

'strong positive correlation between inflation & size of price change'

What does the data say?

Little to no correlation between 'size of price change' and 'rate of inflation' when inflation is low to moderate.

Economists have, hitherto, taken this to mean inflation does not cause significant relative price distortion

A different theoretical lens to view the data

We develop a model in which it is possible to have:

1. Near-zero (or even negative) causal correlation between inflation and 'size of price change'
2. Yet size relative price distortion

Basic setup

There is a finite set of monopolistically competitive firms. We denote the set of firms by $N = \{1, \ldots, n\}$. Each firm i converts inputs to output using a production technology characterized by a Cobb-Douglas function.

$$f_i(x_1, \ldots, x_n) := \prod_{j \in N} x_j^{a_{ij}} \tag{1}$$

a_{ij} is the share of firm i's expenditure on input j. Therefore, $\sum_{j \in N} a_{ij} = 1$.

The adjacency matrix A defines the production network of the economy. $A = (a_{ij})_{i,j \in M}$ such that $a_{ij} \neq 0$ if j is a supplier of i and zero otherwise.

General equilibrium

A general equilibrium of the economy $\mathcal{E}(A)$ is a collection $(\bar{p}_i, \bar{x}_i, \bar{q}_i)_{i\in N} \in R_+^N \times (R_+^N)^N \times R_+^N$ of prices, input flows, and outputs if and only if:

1. Feasibility constraint:
   $\bar{q}_i = \prod_{j\in N} \bar{x}_{ji}^{a_{ij}}, \ \forall i \in N$
2. Firms maximize profits:
   $\bar{x}_{ji} = \frac{a_{ij}\bar{p}_i\bar{q}_i}{\bar{p}_j}, \ \forall i, j \in N$
3. Markets clear:
   $\bar{q}_i = \sum_{j\in M} x_{ij}, \ \forall i \in N$

Existence and uniqueness of equilibrium

Upto price normalization, the economy $\mathcal{E}(A)$ has a unique equilibrium $(\bar{p}_i, \bar{x}_i, \bar{q}_i)_{i \in N} \in R_+^N \times (R_+^N)^N \times R_+^N$, with $\bar{p}_i = \prod_{j \in N} \left(\frac{\bar{p}_j}{a_{ij}}\right)^{a_{ij}}$

If the matrix A characterizing the production network is irreducible and aperiodic, a standard result from the random matrix theory guarantees the existence of a unique equilibrium.

Time sequence of events

Interaction between firms is defined by the following sequences of events.

1. Each firm i receives demand $\sum_{j \in N} \alpha_{ji} m_{j,t}$
2. Given demand $\sum_{j \in N} \alpha_{ji} m_{j,t}$ and output $q_{i,t}$, the market clearing price for firm i is: $p_{i,t} = \frac{\sum_{j \in N} \alpha_{ji} m_{j,t}}{q_{i,t}}$
3. Firms allocate intermediate inputs: $x_{ij}^{t} = \frac{a_{ji} w_j^t}{p_i^t}$
4. Firms produce output using intermediate inputs. This output is sold at the next time step.
5. The money holdings of each firm at the next time step is:$m_{i,t+1} = \sum_{j \in M} a_{ji} m_{j,t}$

Other assumptions about the network

- The network is irreducible, aperiodic, and row-stochastic (yielding a first eigenvalue of $\lambda_1 = 1$)
- The degree distribution of the production network follows a powerlaw $P(d) \propto d^{-\alpha}$ with $\alpha > 1$
- The production network is negatively assortative, with mean supplier degree $\kappa_i = C d_i^{-v}$, with $v \in (0, 1)$
- Second eigenvalue is complex, with the real part being negative

Stability

The system has good stability properties: the rate of convergence to equilibrium is bounded by the subdominant eigenvalue of the adjacency matrix A that characterizes the production network.

More specifically, $\exists z \in R_+$ such that: $||m_t - \bar{m}|| \leq z\rho^t$, where ρ is the second largest Eigenvalue of A.

'Local state-dependent' price stickiness

The probability that firm i resets its price in period T is given by:

$$\eta_{i,T} := g_{u\pi}(\pi_u) \; g_i(\delta_i)$$

with $u := T - \widehat{T}$ waiting time, and $\pi_u := u\pi$ the cumulative inflation since the last reset. $g_{u\pi}$ is a monotone map from *cumulative inflation* since the last reset:

$$g_{u\pi}(0) = 0 \qquad \lim_{u\to\infty} g_{u\pi}(\pi_u) = 1$$

And g_i measures the firm-specific deviation in demand since the last price reset, which is orthogonal to aggregate inflation, with δ_i as the 'excess degree' of a firm. We further assume:

$$g_i(0) = 0 \qquad \lim_{\delta_i\to\infty} g_i(\delta_i) = c_1 \qquad \lim_{\delta_i\to-\infty} g_i(\delta_i) = c_2 \qquad c_1, c_2 \in (0,1)$$

Monetary shocks as heterogeneous cash injections

Definition (Monetary shock)

Let $M_t = \sum_{i \in N} m_{i,t}$ denote the total money in circulation at time t, and let $\pi \in [0,1]$ represent the aggregate size of a monetary shock. A monetary expansion of size π injects πM_t units of money into the economy by updating each firm's wealth according to

$$m_{i,t+1} = m_{i,t} + \pi M_t \frac{(m_{i,t})^\theta}{\sum_{j \in N}(m_{j,t})^\theta}$$

$\theta \in (0,1)$, small firms receive a disproportionately large share of new money

Convergence of injection simplex

- In a separate mathematical paper, we show that the injection operator converges uniformly
- In this paper, we derive the limit simplex
- The limit simplex depends on the degree distribution, injection heterogeneity θ, and assortativity exponent ν

Approach to proofs

- Assume quantity produced is constant
- Demand faced by a firm is the money holdings of its buyers (including new injection) transformed by the adjacency matrix
- The adjacency matrix can be decomposed and approximated as follows:

$$\mathbf{A} = \mathbf{v}_1\mathbf{u}_1^\top + \sum_{j=2}^{n} \lambda_j \mathbf{v_j}\mathbf{u_j}^\top$$
$$\approx \mathbf{v}_1\mathbf{u}_1^\top + \lambda_2\mathbf{v}_2\mathbf{u}_2^\top$$

  wherein $\mathbf{v}_1\mathbf{u}_1^\top$ is the permanent mode and $\lambda_2\mathbf{v}_2\mathbf{u}_2^\top$ is the transitory mode.
- The left and right eigenvectors can be substituted with their moment approximations.
- The time dynamics of demand faced by each firm, therefore, can be reduced to an equation that involves certain moments of the degree distribution, the parameter of injection heterogeneity, and λ_2

Stochasticity

- Monetary shocks are deterministic
- Stochasticity comes from degrees drawn from a powerlaw distribution
- Using LLN and CLT, under reasonable conditions, we are able to assert that variables of interest concentrate around a Gaussian

Average Absolute Price Change ϕ

Definition

Average Absolute Price Change ϕ

Let p_j^t be the price of firm j at time t. Define the log-price change for firm j between periods $t-1$ and t as

$$\Delta_j^t = \log(p_j^t) - \log(p_j^{t-1}).$$

Then the Average Absolute Price Change denoted by ϕ^t is defined as

$$\phi^t = \frac{1}{n}\sum_{j=1}^{n}|\Delta_j^t|.$$

We define the long-run APPC as

$$\phi = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\phi^t.$$

ϕ Calvo and ϕ Menu-Cost

Remark (Average Price Change in Calvo and Menu-Cost Models)

Under steady-state inflation π, the average absolute price change (APPC), denoted by ϕ, is given by

$$\phi_{Calvo} \approx \pi \frac{1-\mu}{\mu}, \quad \text{and} \quad \phi_{MC} \approx \frac{\pi}{\mu},$$

where in the Calvo model, μ is the exogenously given per-period probability of price adjustment, while in the menu-cost model, μ is determined endogenously (depending on the menu cost and rate of inflation).

Note that since $0 < \mu \leq 1$, ϕ_{Calvo} and ϕ_{MC} will be greater than the steady-state inflation π.

Theorem 1: Boundedness of the size of price change

Theorem (Size of price change)

In a network economy under steady inflation π, during the transient, the prices of large firms grow at a rate greater than π, and prices of small firms grow slower than π. Therefore, the growth of a price index that gives disproportionately high weight to small firms will be bounded from above by the rate of inflation:

$$\phi_T \leq \pi,$$

The small-large firm bifurcation is given by $\delta_i := d_i^{\nu^2} - \mathbb{E}[d^{\nu^2}]$

In steady-state, all prices grow at the rate of inflation, and so does the price index.

Glimpse of the proof

- $\Delta p_{i,T} = \log\left(\frac{D_{i,T}}{D_{i,T-1}}\right) = \log\left(\frac{P_{i,T}+S_{i,T}}{P_{i,T-1}+S_{i,T-1}}\right)$
- $S_{i,\mathrm{T}} := \sum_{t=1}^{\mathrm{T}-1} r_{i,t}$
- $r_{i,t} := \lambda_2^{T-t}\mathbf{v}_2\mathbf{u}_2^{\top}\epsilon \approx \delta_i\ \chi_t\ \lambda_2^{T-t}\ \ C_t$

  where:
  $\chi_t = \pi\,(1+\pi)^{\,t-1}$,
  $\delta_i := d_i^{v^2} - \mathbb{E}[d^{v^2}]$
  $C_t := \ \mathbb{E}_{\gamma_t}[d^{-v}] \ - \ \mathbb{E}[d^{-v}]$, with $\mathbb{E}_x[y] := \sum_{j=1}^{N} x_j y_j$,

  (Note that γ_t is the vector of the proportions in which new money is divided among the firms)

Specific result

Let price weights be given by:

$$\mu_i(\zeta) := \frac{d_i^{\zeta}}{\sum_{j=1}^{n} d_j^{\zeta}}$$

Then:

$$\phi_T < \pi \quad \Longleftrightarrow \quad 0 < \zeta < \zeta^*$$

where

$$\zeta^* \simeq \min\{\, 1,\ \alpha - \nu^2 \,\}$$

Supplementary results

- The absolute gap between the average size of price changes and inflation increases with $|\lambda_2|$
- The size of price change in the first period after a monetary shock is smaller than in the steady-state inflation regime
- Independence of ζ^* from θ
- Gaussian concentration of the average size of price change ϕ_T

Corollary 1(a): Responsiveness of size of price change to rate of inflation

The elasticity of 'size of price change' with the 'rate of inflation' can be negative in the early periods of a new inflationary regime. In steady state, the elasticity is positive, though it can be less than one. The elasticity decreases as the network becomes more heavy-tailed and more disassortative, and the monetary shocks become more heterogeneous.

$$\mathcal{L} \approx 1 + C\,K\,\frac{\mathbb{E}[d]}{\mathbb{E}[d^{\zeta}]}\left(\mathbb{E}[d^{\zeta+w-1}] - \mathbb{E}[d^{w}]\,\mathbb{E}[d^{\zeta-1}]\right)$$

where $K = \frac{\lambda_2}{1-\lambda_2(1+\pi)}$

Corollary 1(b): Local state-dependent price stickiness generates near-zero correlation between the rate of inflation and the size of price change

With fully flexible prices:

$$\Delta p_{i,T} \approx \pi + z_i \xi_T$$

where

$$z_i = \frac{\delta_i}{\mu_i}, \qquad \xi_T = \frac{\pi(1+\pi)}{\tilde{m}} \sum_{k=1}^{T-1} C_k \lambda_2^{T-k} (1+\pi)^{k-T}$$

wherein the term $z_i \xi_T$ captures the deviation in a firm's price change from inflation due to the deviation in its size from the mean size.

Corollary 1(b) ..

Now, suppose we write ξ_T as $\frac{\pi(1+\pi)}{\widetilde{m}}\mathcal{X}_T$, where

$$\mathcal{X}_T = \sum_{k=1}^{T-1} C_k \, \lambda_2^{T-k} \, (1+\pi)^{k-T}$$

Note that $\mathcal{X}_T$ reflects the accumulation of past waves of shocks

With sticky prices, ξ_T depends not only on $\mathcal{X}_T$ but on all the $\mathcal{X}_{\widehat{T}<T}$ since the last price adjustment. More specifically, the size of price change of a firm that last changed it price at period $\widehat{T} < T$:

$$\Delta p_{i,T} \;\approx\; \pi + z_i \, \frac{\pi(1+\pi)}{\widetilde{m}} \sum_{L=\widehat{T}}^{T} \sum_{k=1}^{L-1} C_k \, R^{L-k}$$

$$\Delta p_{i,u} \;\approx\; \pi + z_i \, u \, \pi (1+\pi)^2 \, C \, \frac{1}{\widetilde{m}\,(1+\pi-\lambda_2)} + K_{\pi,\lambda_2}$$

where $R := \frac{\lambda_2}{1+\pi}$ and $z_i = \frac{\delta_i}{\mu_i}$

Corollary 1(b) final result

$$\mathbb{E}[\phi] := \sum_{i=1}^{n} \mu_i(\zeta)\, E[\Delta p_i]$$

$$\approx \pi \;+\; \pi\, \mathcal{V}\, H(d, f_v, f_g) \;+\; K_{\pi,\lambda_2}$$

where

$$\mathcal{V} := (1+\pi)^2\, C\, \frac{1}{\widetilde{m}\,(1+\pi-\lambda_2)}$$

$$H(d, f_v, f_g) := \frac{\mathbb{E}\left[d^{\zeta+1}\, f_\nu\, f_g\right]}{\mathbb{E}[d^\zeta]} \quad f_\nu := \frac{d_i^{\nu^2} - \mathbb{E}\left[d^{\nu^2}\right]}{d_i^{\nu^2}} \quad f_g := \frac{1-\widetilde{g}_{u\pi}\, g_i}{\widetilde{g}_{u\pi}\, g_i}$$

Taking the derivative with respect to π tells us that when prices are sticky, the size of price change responds little to changes in rate of inflation

Relative price gap

Definition (Relative Price Gap)

Let $p_{i,t}$ be the price of firm i at period t, and let $p^*_{i,t}$ be the equilibrium price corresponding to the monetary mass at period t. Let the price of firm k be the numeraire for the economy. Then, for any firm i, the log-relative price at period t is: $r_{i,T} := \frac{p_{i,T}}{p_{k,T}}$. And the equilibrium relative price (which does not depend on the monetary mass) is:$r^*_i = \frac{p^*_i}{p^*_k}$. The *Relative Price Gap*, denoted by ω_T, is defined as:

$$\omega_T := \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(r_{i,T} - r^*_i\right)^2}$$

Furthermore, *Steady Relative Price Gap* is defined as the time-average of ω_t: $\omega = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\omega_T$

Relative price entropy

Definition (Relative Price Entropy)

Let the relative price entropy ψ_T be the Kullback-Leibler distance between the distribution of relative prices general equilibrium and in period T. More specifically, normalized relative prices at period T and in general equilibrium be:

$$R_{i,T} = \frac{\mathbb{E}[r_{i,T}]}{\sum_{j=1}^{n} \mathbb{E}[r_{j,T}]}, \qquad R_i^* = \frac{r_i^*}{\sum_{j=1}^{n} r_j^*},$$

Naturally, $\sum_{i=1}^{n} R_{i,T} = 1$ and $= \sum_{i=1}^{n} R_i^*$. Then:

$$\psi_T := \sum_{i=1}^{n} R_{i,T} \log\left(\frac{R_{i,T}}{R_i^*}\right)$$

And

$$\psi \;=\; \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \psi_T$$

Theorem 2: Relative price gap

With fully flexible prices, the steady-state relative price gap is given by:

$$\omega \approx \pi\, k\, \mathcal{A}^2\, \widetilde{H} \sqrt{\mathcal{K}_1 - 2\,\mathcal{K}_2 + \mathcal{B}^2\,\mathcal{K}_3}$$

where $\mathcal{A} := \frac{\alpha}{\alpha - 1}$, $\mathcal{B} := \frac{\alpha}{\alpha - \nu^2}$, $\mathcal{K}_1 := \frac{1}{\alpha + 3 - 2\nu^2}$, $\mathcal{K}_2 := \frac{1}{\alpha + 3 - \nu^2}$, and $\mathcal{K}_3 := \frac{1}{\alpha + 3}$.

And

$$\widetilde{H} = \widetilde{C}_{\mathrm{ub}} \frac{\lambda_2}{1 - \lambda_2} \qquad \widetilde{C}_{\mathrm{ub}} \approx \frac{\alpha - (1 - v)\,\theta}{\alpha - (1 - v)\,\theta + v} - \frac{\alpha}{\alpha + v}$$

with k is a strictly positive constant.

Supplementary results

- Fat-tails in the degree distribution increase relative price distortion
- Negative assortativity increases relative price distortion
- Heterogeneity in the initial impact of monetary shocks increases relative price distortions
- Rate of convergence to equilibrium decreases relative price distortions
- When network is random, ω_T has Gaussian concentration

Glimpse of proof

- Choose as numéraire the price of firm k, where k is defined by

$$k = \arg\min_{1 \le j \le n} \left(\mathbb{E}[d^{\nu^2}] - d_j^{\nu^2} \right)^2,$$

- Write replace quantity with a function of degree, which reduces prices to network attributes
- Relative price of firm i becomes

$$r_{i,T} = \frac{\mu_i + \delta_i \pi \sum_{t=1}^{T-1} \left(\frac{\lambda_2}{1+\pi} \right)^{T-t} C_t}{\mu_k + \delta_k \pi \sum_{t=1}^{T-1} \left(\frac{\lambda_2}{1+\pi} \right)^{T-t} C_t} \; \frac{h(d_k)}{h(d_i)}$$

which yields a simple form when $\delta_k = 0$

Corollary 2(a)

With fully flexible prices, the elasticity of the relative price gap to the rate of inflation in the steady state is approximately 1

Corollary 2(b)

With local state-dependent price stickiness

- Expected relative prices do not depend on price stickiness in the steady state
- In the transient, relative price gap increases with price stickiness
- In the transient, relative price entropy has a non-monotonic relation with price stickiness

Glimpse of proof

For any period $K > 0$, a firm may change its price at K, whether or not it has changed price previously. However, the probability of changing price at K depends on the history of prior changes, as the hazard function $g_{u\pi}$ increases with the waiting period. Let ϑ_K denote the probability that firm i changes its price at period K. We define:

$$\vartheta_{i,0} := 1, \qquad \vartheta_{i,1} := g_{\pi} g_i \qquad \vartheta_{i,2} := (1 - \vartheta_1)\, g_{2\pi} g_i + \vartheta_{i,1}\, g_{\pi} g_i$$

More generally, for any $T \geq 1$,

$$\vartheta_{i,T} = \sum_{u=1}^{T} \left[\prod_{v=1}^{u-1} (1 - g_{v\pi} g_i) \right] g_{u\pi}\, g_i$$

This expression accounts for all possible paths by which the firm could arrive at a price change in period T, after zero or more periods of inaction.
Let $\Theta_{\widehat{T}}$ denote the probability that firm i, at period T, is charging the market-clearing price from period $\widehat{T} < T$. Then:

$$\Theta_{i,\widehat{T}} := \vartheta_{i,\widehat{T}} \cdot \prod_{S=\widehat{T}+1}^{T} (1 - \vartheta_{i,S})$$

Glimpse of proof ..

- Now that we know the probability of charging the market clearing price from period $\widehat{T}$, all we need the 'market clearing price' itself to compute expectation
- From the flexible price results we know that

$$p_{i,\widehat{T}} = \frac{1}{h(d_i)}\,\tilde{m}\,(1+\pi)^{\widehat{T}-1}\Big(\mu_i + \delta_i\,\pi \sum_{t=1}^{\widehat{T}-1}\big(\tfrac{\lambda_2}{1+\pi}\big)^{\widehat{T}-t} C_t\Big)$$

- We can use these to compute expected relative price and compare it to actual relative price to derive relative price gap and relative price entropy

Principle result

- Sizeable relative price distortions even when prices are fully flexible
- Price stickiness creates near-zero correlation between size of price change and rate of inflation, but does not dampen relative price distortions
- With Calvo and menu-cost pricing, the size of the price change is a good proxy for relative distortions. Not true in a network economy model with local state-dependent pricing.

Monetary theory and networks research program

- Doctoral dissertation: some counter-intuitive facts about behavior of the distribution of price
- Postdoctoral work: counter-intuitive behavior of price-level itself (250-year old puzzle)
- This paper: strange joint-behavior of size of price change and relative price deviation, with implications for the welfare cost of inflation

Thank you for the opportunity to present this paper at your conference.